On the Lupaş $q$-analogue of the Bernstein operator

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The importance of the Bernstein polynomials opened the gates to the discovery of their numerous generalizations and applications in various mathematical disciplines. As an example, recent generalizations based on the $q$-integers emerged due to the speedy development of the $q$-calculus. A. Lupaş was the person who pioneered work on the $q$-versions of the Bernstein polynomials. In 1987, he introduced (cf. [1]) a $q$-analogue of the Bernstein operator, and investigated its approximation and shape-preserving properties. Since then, the study of the $q$-analogue has been in progress, see [2]-[4].

Let $q > 0$. For any $n = 0, 1, 2, \ldots$ the $q$-integer $[n]_q$ is defined by:

$$[n]_q := 1 + q + \cdots + q^{n-1} \quad (n = 1, 2, \ldots), \quad [0]_q := 0;$$

and the $q$-factorial $[n]_q!$ by:

$$[n]_q! := [1]_q[2]_q \cdots [n]_q \quad (n = 1, 2, \ldots), \quad [0]_q! := 1.$$

For integers $0 \leq k \leq n$, the $q$-binomial coefficient is defined by:

$$\left[ \begin{array}{c} n \\ k \end{array} \right]_q := \frac{[n]_q!}{[k]_q! [n-k]_q!}.$$  

**Definition 1.** Let $q > 0$, $f : [0, 1] \to \mathbb{C}$. The linear operator

$$(R_{n,q} f)(z) := \sum_{k=0}^{n} f \left( \frac{[k]_q}{[n]_q} \right) b_{nk}(q; x), \quad n \in \mathbb{N},$$

where

$$b_{nk}(q; x) := \left[ \begin{array}{c} n \\ k \end{array} \right]_q \frac{q^{k(k-1)/2} x^{k} (1-x)^{n-k}}{(1-x+qx) \cdots (1-x+q^{n-1}x)}, \quad k = 0, \ldots, n$$

is called the Lupaş $q$-analogue of the Bernstein operator.

Clearly, if $q = 1$, then $R_{n,q}$ reduce to the classical Bernstein polynomials. In the case $q \neq 1$, operators $R_{n,q}$ are rational functions rather than polynomials.
The limit $q$-Lupaş operator $(\Lambda_q f)$ comes out naturally as a limit for a sequence of the Lupaş $q$-analogues of the Bernstein operator. Lately, it has been studied by several authors from different perspectives of mathematical analysis and approximation theory. This operator is closely related to the $q$-deformed Poisson probability distribution, which is used widely in the $q$-boson operator calculus.

In this talk, we discuss the convergence properties of the operators $R_{n,q} f$ as well as some analytic properties of the $(\Lambda_q f)(z)$. In particular, we examine the conditions under which $\Lambda_q f$ can be either an entire function or a rational one.

References


